

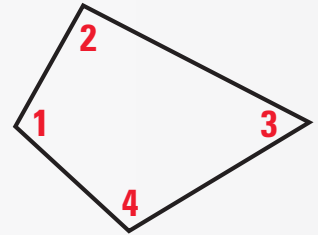
- I can define a polygon and identify polygons.
- I can define a diagonal.
- I can compare and contrast convex and concave polygons.
- I can define and solve problems with regular polygons.
- I can apply the quadrilateral sum theorem.

THEOREM

THEOREM 6.1 *Interior Angles of a Quadrilateral*

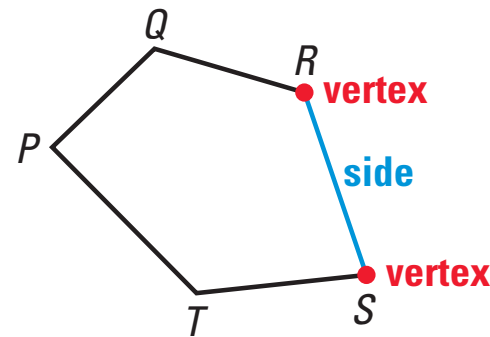
The sum of the measures of the interior angles of a quadrilateral is 360° .

$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 = 360^\circ$$



A **polygon** is a plane figure that meets the following conditions.

1. It is formed by three or more segments called **sides**, such that no two sides with a common endpoint are collinear.

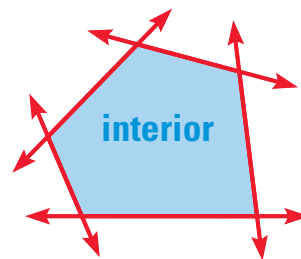


2. Each side intersects exactly two other sides, one at each endpoint.

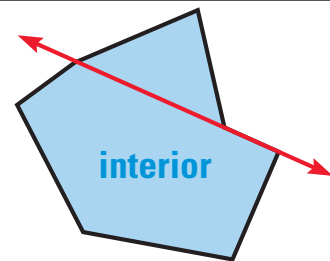
Each endpoint of a side is a **vertex** of the polygon. The plural of *vertex* is *vertices*. You can name a polygon by listing its vertices *consecutively*. For instance, *PQRST* and *QPTSR* are two correct names for the polygon above.

A polygon is **convex** if no line that contains a side of the polygon contains a point in the interior of the polygon.

A polygon that is not convex is called **nonconvex** or **concave**.



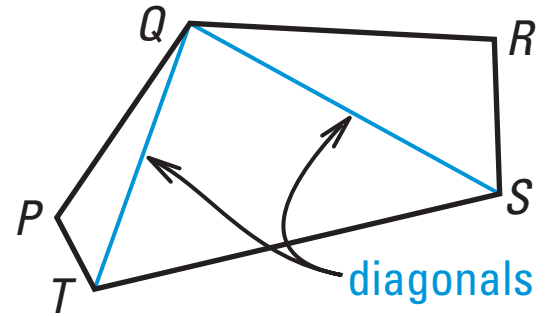
convex polygon



concave polygon

A polygon is **equilateral** if all of its sides are congruent. A polygon is **equiangular** if all of its interior angles are congruent. A polygon is **regular** if it is equilateral and equiangular.

A **diagonal** of a polygon is a segment that joins two *nonconsecutive* vertices. Polygon $PQRST$ has 2 diagonals from point Q , \overline{QT} and \overline{QS} .



STUDENT HELP

Study Tip

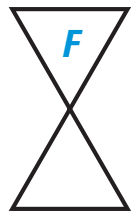
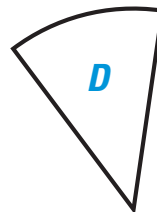
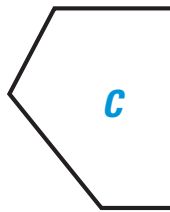
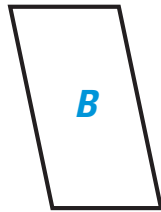
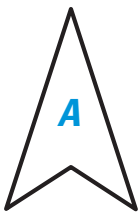
Two vertices that are endpoints of the same side are called *consecutive vertices*. For example, P and Q are consecutive vertices.

Number of sides	Type of polygon
3	Triangle
4	Quadrilateral
5	Pentagon
6	Hexagon
7	Heptagon

Number of sides	Type of polygon
8	Octagon
9	Nonagon
10	Decagon
12	Dodecagon
n	n -gon

EXAMPLE 1 *Identifying Polygons*

State whether the figure is a polygon. If it is not, explain why.



SOLUTION

Figures A , B , and C are polygons.

- Figure D is *not* a polygon because it has a side that is not a segment.
- Figure E is *not* a polygon because two of the sides intersect only one other side.
- Figure F is *not* a polygon because some of its sides intersect more than two other sides.

Polygons

_____ **Example:** Identify the figures that are not simple polygons.

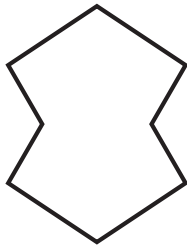


EXAMPLE 2

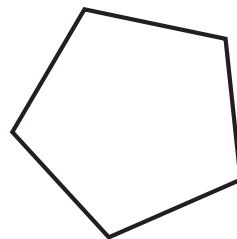
Identifying Convex and Concave Polygons

Identify the polygon and state whether it is convex or concave.

a.

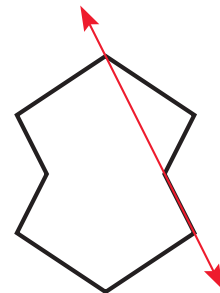


b.

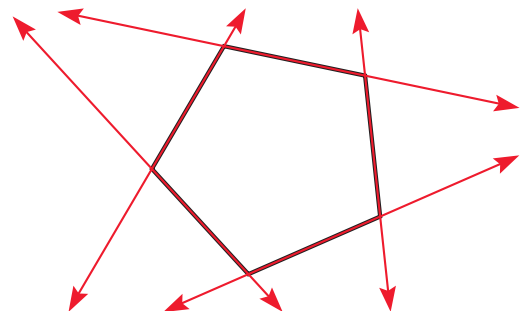


SOLUTION

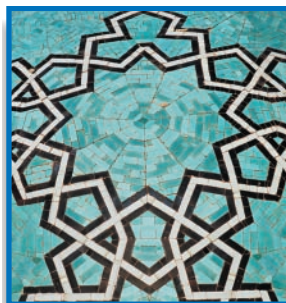
- a. The polygon has 8 sides, so it is an octagon. When extended, some of the sides intersect the interior, so the polygon is concave.



- b. The polygon has 5 sides, so it is a pentagon. When extended, none of the sides intersect the interior, so the polygon is convex.



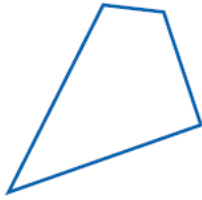
.....



This tile pattern in Iran contains both convex and concave polygons.

Identify the polygon and state whether it is convex or concave.

2.



3.

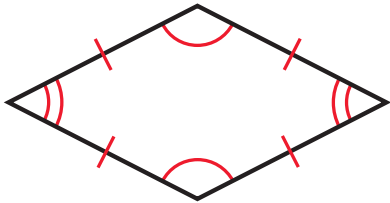


EXAMPLE 3

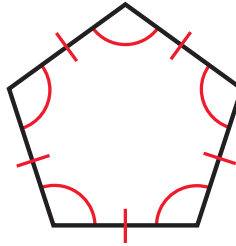
Identifying Regular Polygons

Decide whether the polygon is regular.

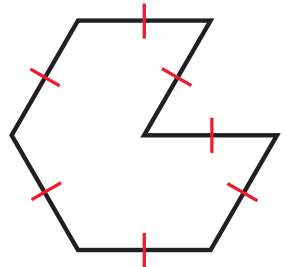
a.



b.



c.



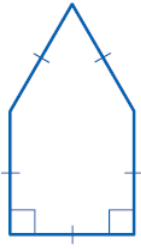
SOLUTION

- a. The polygon is an equilateral quadrilateral, but not equiangular. So, it is not a regular polygon.
- b. This pentagon is equilateral and equiangular. So, it is a regular polygon.
- c. This heptagon is equilateral, but not equiangular. So, it is not regular.

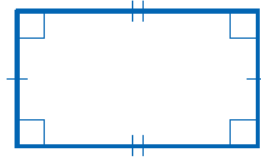


Decide whether the polygon is regular.

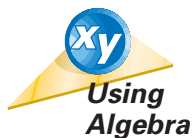
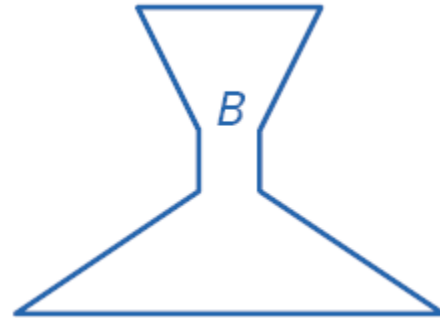
4.



5.



6. State whether the figure is a polygon. If it is, state whether it is convex or concave. If it is not a polygon, explain why not.



EXAMPLE 4 Interior Angles of a Quadrilateral

Find $m\angle Q$ and $m\angle R$.

SOLUTION

Find the value of x . Use the sum of the measures of the interior angles to write an equation involving x . Then, solve the equation.

$$x^\circ + 2x^\circ + 70^\circ + 80^\circ = 360^\circ$$

$$3x + 150 = 360$$

$$3x = 210$$

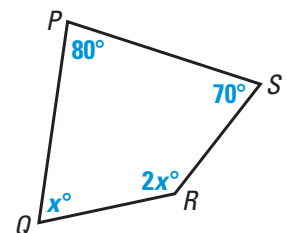
$$x = 70$$

Sum of measures of int. \angle s of a quad. is 360° .

Combine like terms.

Subtract 150 from each side.

Divide each side by 3.



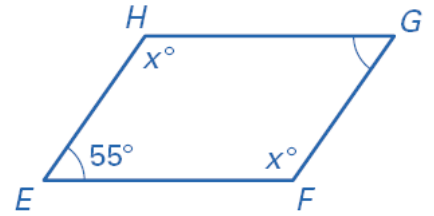
Find $m\angle Q$ and $m\angle R$.

$$m\angle Q = x^\circ = 70^\circ$$

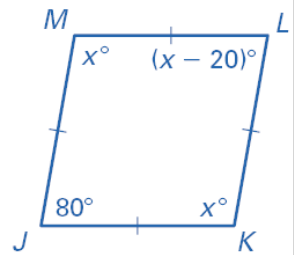
$$m\angle R = 2x^\circ = 140^\circ$$

► So, $m\angle Q = 70^\circ$ and $m\angle R = 140^\circ$.

7. Example: Find $m\angle F$, $m\angle G$, & $m\angle H$.



8. Find $m\angle K$, $m\angle L$, & $m\angle M$. Is quadrilateral JKLM regular?



9. What is the plural of vertex?

10. What do you call a polygon with 8 sides? a polygon with 15 sides?

